makes k_t about twice too great; and, secondly, because T_1/T_2 measures the ratio of the extreme temperatures in the separator, and not the ratio of the mean temperatures of the two halves of the gas.

A point of special interest in Mr. Ibbs' paper is the shortness of the time, about three minutes, required for thermal diffusion to take effect.

The Dynamics of Revolving Fluid on a Rotating Earth.

By Capt. D. Brunt, M.A., B.Sc.

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In a paper on the dynamics of revolving fluids, the late Lord Rayleigh* considered the special case of fluid revolving about a fixed vertical axis, neglecting the rotation of the earth. The object of the present paper is to investigate the modifications of Rayleigh's results which are brought about by the rotation of the earth, and by translation in a vertical plane of the axis of symmetry. Air is treated as an incompressible non-viscous fluid.

Let the motion be referred to rectangular axes, x, y, z, rotating with the earth, the axis of z being vertical and the axes of x and y in the horizontal plane. At a point x, y, z, the components of velocity are u, v, w, the pressure is p, and density ρ. If gravity is the only impressed force, the equations of motion are

$$\mathrm{D}u/\mathrm{D}t = -1/\rho\, dp/dx + lv, \tag{1}$$

$$\mathrm{D}v/\mathrm{D}t = -1/\rho\, dp/dy - lu, \tag{2}$$

$$\mathrm{D}w/\mathrm{D}t = -1/\rho\, dp/dz + g, \tag{3}$$

where $$\mathrm{D}/\mathrm{D}t = d/dt + u\, d/dx + v\, d/dy + w\, d/dz$$

and the equation of continuity is

$$du/dx + dv/dy + dw/dz = 0, \tag{4}$$

where $l = 2\omega \sin\phi$, ω being the angular velocity of rotation of the earth, and ϕ the latitude. The terms lv, $-lu$, in the first two equations represent components of the deviating force due to the earth's rotation, and the inclusion of these terms takes complete account of the rotation of the earth.

If values of u, v, w, p, be found to satisfy equations (1) to (4), then if U be a constant, $u + \mathrm{U}$, v, w, $p - \rho l \mathrm{U} y$ will also satisfy these equations. In other words, to any system of velocities which satisfies the equations there may be added a uniform velocity of translation U, provided the pressure distribution be corrected by the addition of a horizontal gradient of pressure of amount $2\omega\rho \mathrm{U}\sin\phi$ at right angles to U. This affords a simple method of proceeding from the case of fluid rotating about a fixed axis to the case where the axis has a uniform velocity of translation, and we may therefore restrict the discussion, in the first place, to the case of motion symmetrical about a fixed axis.

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* 'Roy. Soc. Proc.,' A, vol. 93, p. 148 (1916).

It is convenient to refer the motion to cylindrical co-ordinates, r, θ, z, the velocities u, v, w being reckoned positive in the directions of r, θ, z, increasing. It is assumed that the motion is symmetrical about the axis of z. The only impressed force being gravity, the equations of motion are

$$du/dt+u\, du/dr-v^2/r+w\, du/dz = -1/\rho\, dp/dr+lv, \tag{5}$$

$$dv/dt+u\, dv/dr+uv/r+w\, dv/dz = \qquad\qquad -lu, \tag{6}$$

$$dw/dt+u\, dw/dr \qquad\quad +w\, dw/dz = -1/\rho\, dp/dz+g. \tag{7}$$

In the special case where $u = w = 0$, equation (5) reduces to the ordinary equation for the gradient wind for circular isobars.

The case where $w = 0$, and u and v are functions of r and t only, is the most interesting case considered by Rayleigh. The motion is then two-dimensional, and he expresses it conveniently by the vorticity ζ, which moves with the fluid, and the stream-line function ψ, which are connected by the equation

$$1/r\, d/dr\, (r\, d\psi/dr)+1/r^2\, d^2\psi/d\theta^2 = 2\zeta, \tag{8}$$

and

$$u = -d\psi/rd\theta, \tag{9}$$

$$v = d\psi/dr. \tag{10}$$

Since u is independent of θ, $d^2\psi/d\theta^2 = 0$, and

$$1/r\, d/dr\, (r\, d\psi/dr) = 2\zeta.$$

The solution of these two equations therefore yields

$$\psi = 2\int dr/r \int \zeta r\, dr+\mathrm{A}\log r+\mathrm{B}\theta, \tag{11}$$

where A and B are independent of r and θ, but may vary with time t. We then have

$$u = -\mathrm{B}/r, \tag{12}$$

$$v = 2/r\int \zeta r\, dr+\mathrm{A}/r. \tag{13}$$

In the case where the fluid has an initial rotation as a solid body (with angular velocity ζ) there is a ζ initially, and therefore permanently, constant throughout the fluid, and equation (13) reduces to

$$v = \zeta r+\mathrm{A}/r. \tag{14}$$

The effect of convergence towards the axis is therefore to superpose upon the rotation as a solid body a simple vortex of intensity, A.

Equations (8) to (14) are given by Rayleigh, and are independent of any consideration of the rotation of the earth. The rotation of the earth only affects the determination of the value of A at any given time.

Equation (6) may be written

$$(d/dt+u\,d/dr+w\,d/dz)\,(rv) = -lru = l\mathrm{B}. \qquad (15)$$

This equation expresses the rate of change of the product rv for a particular ring of particles.

The rotating fluid may be supposed to be contained within a cylindrical wall, which moves inward as the fluid converges, fluid being removed along the axis, without disturbing the horizontal motion. Let R_0 be the radius of the wall at time $t = 0$ and R the radius at time t. It has been supposed that initially the fluid rotates as a solid body, so that initially $\mathrm{A} = 0$.

Equation $u = -\mathrm{B}/r$ may be written

$$r\,dr/dt = -\mathrm{B},$$

and on integration this yields

$$\int_0^t \mathrm{B}\,dt = -\tfrac{1}{2}\Big[r^2\Big]_0^t = \tfrac{1}{2}(\mathrm{R}_0{}^2-\mathrm{R}^2). \qquad (16)$$

Integrating equation (15) and substituting from (16) we find

$$(rv)_t-(rv)_0 = l\int_0^t \mathrm{B}dt = \tfrac{1}{2}\,l\,(\mathrm{R}_0{}^2-\mathrm{R}^2). \qquad (17)$$

This equation could also have been derived from the constancy of the circulation round a circuit made up of the same particles. Taking account of the fact that the xy plane has a rotation $\omega \sin\phi$ about the axis of z, the circulation round a circuit of radius r is $2\pi(rv+r^2\,\omega\sin\phi)$, whence equation (17) immediately follows.

But from equation (14), we have, for the boundary,

$$(rv)_0 = \zeta\mathrm{R}_0{}^2, \qquad (rv)_t = \zeta\mathrm{R}^2+\mathrm{A},$$

$$\therefore \qquad \mathrm{A} = (rv)_t-(rv)_0+\zeta(\mathrm{R}_0{}^2-\mathrm{R}^2)$$

$$= (\zeta+\tfrac{1}{2}l)(\mathrm{R}_0{}^2-\mathrm{R}^2),$$

$$\mathrm{A} = (\zeta+\omega\sin\phi)\,(\mathrm{R}_0{}^2-\mathrm{R}^2). \qquad (18)$$

It follows from (14) that

$$v = \zeta r+\frac{(\zeta+\omega\sin\phi)\,(\mathrm{R}_0{}^2-\mathrm{R}^2)}{r}. \qquad (19)$$

Equation (19) differs from Rayleigh's* equation (19) only by the inclusion of the term $\omega\sin\phi$, which represents the effect of the rotation of the earth.

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* *Loc. cit.*, p. 153.

The effect of the earth's rotation is to increase the strength of the superposed simple vortex. It is clear that, when there is originally no rotation, or $\zeta = 0$, the effect of convergence due to the removal of air is to produce a simple vortex, in which the distribution of velocity is given by

$$v = \frac{\omega \sin\phi \, (\mathrm{R}_0{}^2 - \mathrm{R}^2)}{r}. \tag{20}$$

This equation thus gives the effect of the removal of air in a region where there is no vorticity, in contrast with equation (19), which shows the effect in a region having originally uniform vorticity, ζ.

If a cyclone starts as a uniformly rotating disc of radius R_0, with uniform angular velocity ζ, the effect of removal of fluid along the axis is to cause the outer radius R_0 to shrink to a smaller value, R, and, within the region bounded by the cylinder of radius R, to superpose upon the uniform rotation a simple vortex whose intensity increases steadily as the removal proceeds (equation (19)). Outside the cylinder of radius R, the change of motion caused by convergence is represented by a simple vortex, in accordance with equation (20).

The amount of fluid which must be removed from a layer of the vortex of unit thickness, in unit time, is $-2\pi r u$ or $2\pi\mathrm{B}$. In a time t the amount of fluid removed is $2\pi\int_0^t \mathrm{B}\,dt$ or $\pi(\mathrm{R}_0{}^2 - \mathrm{R}^2)$, by equation (17), showing that the intensity of the simple vortex produced by the convergence is directly proportional to the total amount of fluid removed from the region of the axis, and *vice versâ*, the removal of that amount of fluid implies the development of the vortex.

The distribution of pressure is evaluated by the integration of equation (5). If at any time the u motion ceases, the subsequent distribution of pressure is obtained by integrating

$$1/\rho \; dp/dr = v^2/r + lv = v^2/r + 2\,\omega \sin\phi \, v,$$

or, from (19)

$$1/\rho \; dp/dr = \zeta(\zeta + 2\omega\sin\phi)\,r + \frac{2(\zeta + \omega\sin\phi)^2(\mathrm{R}_0{}^2 - \mathrm{R}^2)}{r} + \frac{(\zeta + \omega\sin\phi)^2(\mathrm{R}_0{}^2 - \mathrm{R}^2)^2}{r^3},$$

whence, neglecting the variations of ϕ,

$$p/\rho = \tfrac{1}{2}\zeta(\zeta + 2\omega\sin\phi)\,r^2 + 2(\zeta + \omega\sin\phi)^2(\mathrm{R}_0{}^2 - \mathrm{R}^2)\log r - \tfrac{1}{2}\frac{(\zeta + \omega\sin\phi)^2(\mathrm{R}_0{}^2 - \mathrm{R}^2)^2}{r^2} + \text{constant}. \tag{21}$$

The effect of the inward motion upon the distribution of pressure, while the inward motion is still in progress, is easily evaluated by substitution of $u = -B/r$ in equation (5). Neglecting $w\, du/dz$, we find

$$1/\rho\ dp/dr = v^2/r + lv + B^2/r^3 + 1/r\ dB/dt,$$

which, on integration, yields the same result as equation (21), with the addition of $-\frac{1}{2} B^2/r^2 + dB/dt \log r$ to the right-hand side. The second of the additional terms vanishes if the rate of removal of fluid is constant.

The results derived above may be applied to the supply of air at the axis, involving a progressive increase of the extreme radius R of the disc of fluid concerned. The dynamical effect is to superpose on the original distribution of velocity a simple vortex of negative intensity, or a vortex rotating clockwise. A region of descending air should therefore, merely on account of its descent, acquire a clockwise rotation, with velocity inversely proportional to the distance from the centre of the axis of revolution. If one deals with a stratum of unit thickness, the added air might come from above or below. It is only a question of there being an addition of air to the layer.

If the fluid is added at the centre of the disc, so that the outer radius R_0 increases to R at time t, while the added fluid is bounded by a circle of radius R' at time t, equation (19) still holds within the limits $R' < r < R$, and equation (20) still holds for $r > R$. But R is now greater than R_0, so that the simple whirl produced by divergence from the axis rotates clockwise. Within the added fluid, where $r < R'$, it follows from the constancy of the circulation round a circle of particles (initially zero when the fluid is added at $r = 0$), that

$$rv + r^2\omega \sin\phi = 0,$$

or

$$v = -r\omega \sin\phi. \tag{22}$$

In other words, the added fluid forms a core rotating clockwise as a solid, with angular velocity $\omega \sin\phi$.

The stream-lines of instantaneous flow in the general case, where the distribution of velocity is represented by

$$u = -B/r, \qquad v = \zeta r + A/r$$

are obtained by integrating the equation

$$r d\theta/dr = v/u = -(\zeta r^2 + A)/B. \tag{23}$$

This yields on integration

$$re^{\zeta r^2/2A} = Ce^{-B/A\,\theta}, \tag{24}$$

and the stream-lines are spirals whose angle of convergence increases towards the centre, except in the special case where $\zeta = 0$, and the motion is equivalent to a simple vortex, in which case the stream-lines are equiangular spirals.

In the complex case, where the distribution of velocity is a simple vortex superposed upon solid rotation, the angle of convergence of the horizontal projection of the spiral paths cannot be constant over the whole area unless air is removed over the whole area. The relation vr = constant is the only one which is consistent with air passing horizontally across the area, with no extraction vertically except at the centre.

In all that precedes, the centre of the two-dimensional whirl has been assumed at rest. If the whirl is given a uniform velocity of translation, U, the distribution of velocity relative to the centre remains unaltered, and the spiral paths of instantaneous flow relative to the centre of the whirl are unaltered. All that is necessary is to add a correcting term, $-2\omega \sin \phi \,.\, \mathrm{U}y$ to the equation (21) for the distribution of pressure.

Some Experiments on the Catalytic Reduction of Ethylene to Ethane.

By Dorothy Muriel Palmer, Girton College, Cambridge, and William George Palmer, St. John's College, Cambridge.

(Communicated by Sir Wm. Pope, F.R.S. Received April 14, 1921.)

Finely divided reduced nickel has been applied as an active agent in the catalysis of a great range of reactions, but very little is known about the mechanism of its action. It was thought that the intensive study of some particular process might be of interest, in view of the paucity of quantitative information about catalysis.

The reaction selected for investigation was the combination of ethylene and hydrogen to form ethane. Sabatier* states that in the presence of freshly reduced nickel this reaction begins at 30°–45° C., and continues with the evolution of heat; thus there are few of the difficulties encountered when working at higher temperatures. Ethylene and hydrogen can both be obtained in a high degree of purity, and since the reacting and resultant substances are gases with simple volume relations between them, it was found possible to watch the progress of the reaction by a static method.

This was done by observing the pressure changes produced in a closed vessel containing the catalyst, together with a mixture of ethylene and hydrogen of known composition.

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* 'Comptes Rendus,' vol. 124, p. 1360.